Radiative transfer at relativistic shocks

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ABSTRACT

GRB is one of the phenomena in which radiation transfer occurs in moving media. As a first step to multidimensional numerical calculations of the jet structure we consider 1D special relativistic radiation hydrodynamics by solving the Boltzmann equation for radiative transfer. The structure of relativistic radiative shock is calculated for a number of shock tube problems including strong shock wave, relativistic and radiation dominated cases. Calculations are performed using iteration technique which consistently solves the equations of hydrodynamics and comoving radiative transfer. Qualitative analysis of moment equations for radiation is performed and conditions for the existence of viscous jump are investigated numerically.

INTRODUCTION

The problem of radiation transfer in the relativistic moving media in the most common statement is reduced to the solution of the system of relativistic radiation hydrodynamics equations coupled with relativistic radiation transfer equation. We resolve one of these non-linear problems – the problem of structure of shock wave coupled with radiation at arbitrary velocity of the matter.

There are many paper related to this topic (Imshennik, Morozov, 1964), but most of them are related to non-relativistic consideration (v/c<<1). A number of papers has been published for relativistic velocities of the matter (Farris et al., 2008), but all of them are based on radiation moments approach. In some cases radiation moments approach provides a relevant solution, but sometimes it is difficult to estimate the relevance of the closure condition and the exact solution of kinetic equation is desirable.

In this work we consider 1D plane and stationary shock wave. Due to complexity of the problem a number of simplified assumptions are introduced:

1. Local thermodynamic equilibrium (LTE)
2. The equation of state is taken for the ideal gas

PROBLEM STATEMENT

The complete system of equations consists of the equation of state, the equations of radiation hydrodynamics and the equation of radiation transfer.

Equation of state:

$$P = (\Gamma - 1)\rho_0\epsilon$$

Radiation HD Equations (see Farris et al., 2008 for details):

$$\begin{aligned}
\rho_o u^x &= U_1 \\
\left[\rho_0 + (n+1)P\right] u^x u^0 = U_2 - U_4 &\equiv U_a \\
\left[\rho_0 + (n+1)P\right] (u^x)^2 + P = U_3 - U_5 &\equiv U_b \\
\frac{4}{3}E u^x u^0 + u^x F^0 + u^0 F^x &= U_4 \\
\frac{4}{3}E (u^x)^2 + \frac{1}{3}E + 2u^x F^x &= U_5
\end{aligned}$$

Comoving RT equation (see e.g., Mihalas, 1980)

$$\begin{aligned}
&\frac{\gamma}{c}(1+\beta\mu)\frac{\partial I(\mu,\nu)}{\partial t} + \gamma(\mu+\beta)\frac{\partial I(\mu,\nu)}{\partial r} + \gamma(1-\mu^2)\times \\
&\times\left[\frac{(1+\beta\mu)}{r} - \frac{\gamma^2}{c}(1+\beta\mu)\frac{\partial\beta}{\partial t} - \gamma^2(\mu+\beta)\frac{\partial\beta}{\partial r}\right]\times \\
&\times\frac{\partial I(\mu,\nu)}{\partial\mu} - \gamma\left[\frac{\beta(1-\mu^2)}{r} + \frac{\gamma^2}{c}(1+\beta\mu)\frac{\partial\beta}{\partial t} + \right. \\
&\left. +\gamma^2\mu(\mu+\beta)\frac{\partial\beta}{\partial r}\right]\nu\frac{\partial I(\mu,\nu)}{\partial\nu} + 3\gamma\left[\frac{\beta(1-\mu^2)}{r} + \right. \\
&\left. +\frac{\gamma^2\mu}{c}(1+\beta\mu)\frac{\partial\beta}{\partial t} + \gamma^2(\mu+\beta)\frac{\partial\beta}{\partial r}\right] I(\mu,\nu) = \\
&= \eta(\nu) - \chi(\nu) I(\mu,\nu)
\end{aligned}$$

The equations of hydrodynamics and radiation transfer are solved numerically by iteration scheme.

NUMERICAL SOLUTION

To compare coupling of hydrodynamics equations with radiation transfer equations and different closure conditions we use a number of shock tube configurations described by Farris et al., 2008. Here we consider mildly relativistic cases of strong shock and radiation-pressure dominated shock with the following parameters:

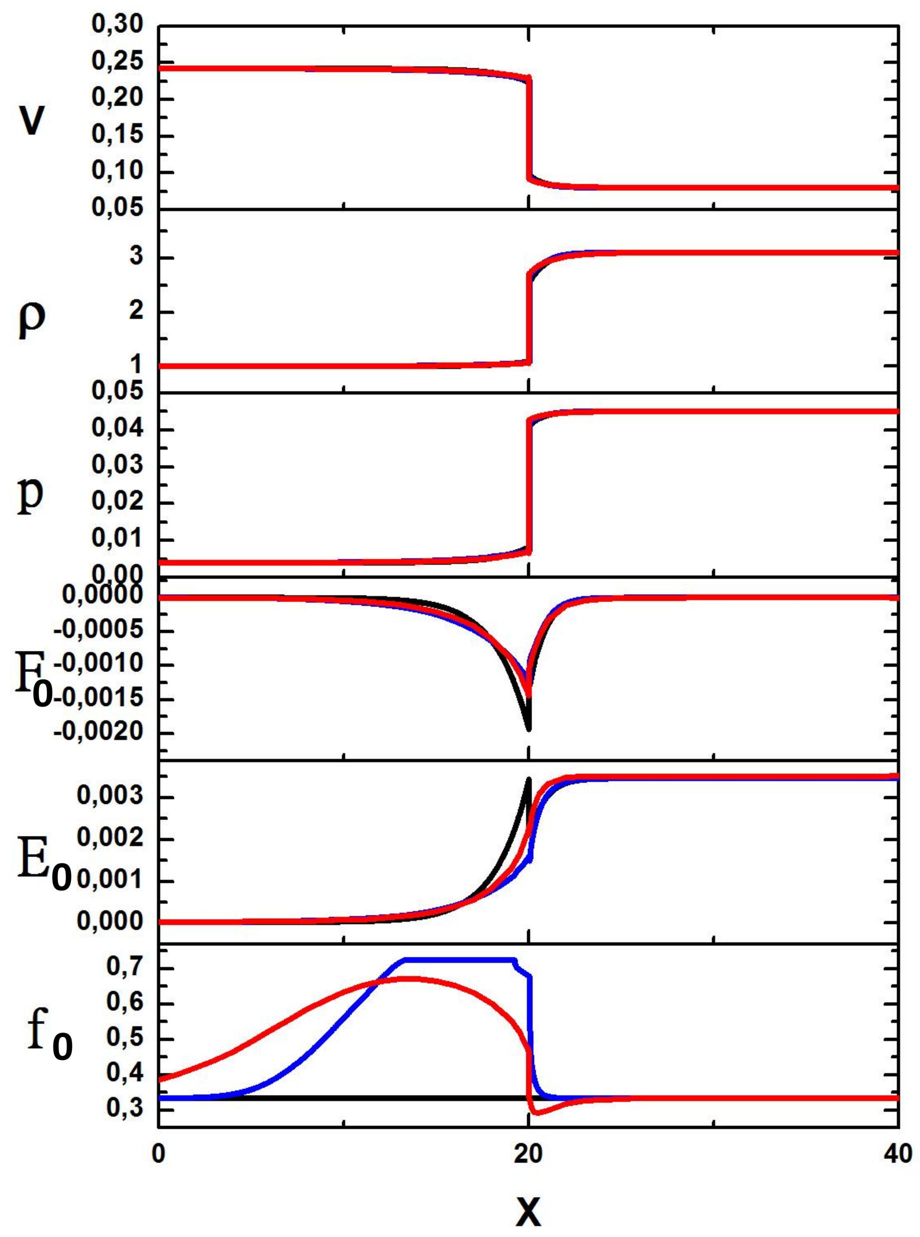
Test	Γ	κ^a	Left state[c]	Right State[c]
1	5/3	0.2	$\rho_0 = 1.0$ $P = 4.0\times10^{-3}$ $u^x = 0.25$ $E = 2.0\times10^{-5}$	$\rho_0 = 3.11$ $P = 0.04512$ $u^x = 0.0804$ $E = 3.46\times10^{-3}$
2	5/3	0.08	$\rho_0 = 1.0$ $P = 6.0\times10^{-3}$ $u^x = 0.69$ $E = 0.18$	$\rho_0 = 3.65$ $P = 3.59\times10^{-2}$ $u^x = 0.189$ $E = 1.30$

- Black line – Eddington approximation (f = 1/3)
- Blue line – M1-closure (Levermore, 1984) , f = f(E,F) combines "optically thin" and "thick" cases
- Red line - Photon Boltzmann equation

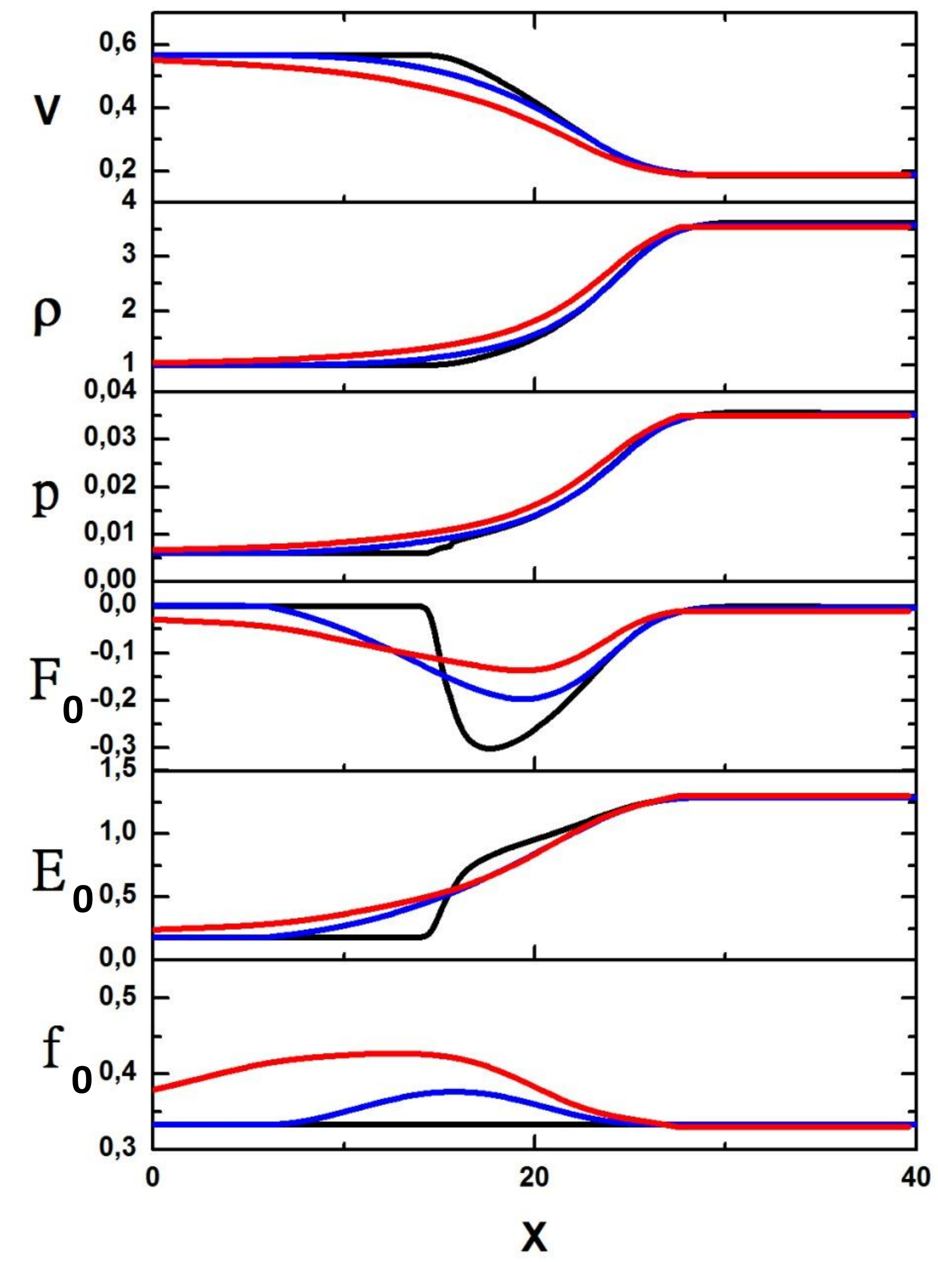
V, ρ, p – velocity, density and pressure of the gas

F_0, E_0, f_0 – comoving radiation energy flux, radiation energy density and Eddington factor $f_0 = P_0/E_0$

1. **Mildly relativistic strong shock**



2. **Radiation-pressure dominated, mildly relativistic shock**



QUALITATIVE ANALYSIS

Let us consider the system of moment equations and perform the phase analyses in FP plane. The differential equations

$$\frac{\partial F}{\partial x} = \frac{\kappa U_1}{\beta}\left(aT^4 + \frac{\beta F(2-f-\beta^2) - P(1-f\beta^2)}{f-\beta^2}\right),$$

$$\frac{\partial P}{\partial x} = \frac{\kappa U_1}{\beta}\left((aT^4 + P)\beta - F\right)$$

$$T = \frac{p}{\rho} = \left(\frac{U_2 - \gamma U_1 - F}{U_1\gamma(n+1)}\right)$$

can be written in the form of linear expansion.

$$dF/dx = a(F_j,P_j)(F-F_j)+b(F_j,P_j)(P-Pj)$$

$$dP/dx = c(F_j,P_j)(F-F_j)+d(F_j,P_j)(P-P_j)$$

Sign of the determinant P=ad-bc determines the nature of the singular points F_j,P_j. If P < 0, we have sadle point, P>0 – node ($(a+d)^2 > P$) or focus ($(a+d)^2 < P$).

Sadle singular points can not be connected by one separatrix (Andronov et al., 1987) and shooting method is required for analyses.

Imshennik & Morozov (1964) analysed various options for transfer equation (simple diffusion equation and the Eddington approximation in the moving medium). Plasma upstream the shock front was taken to be cold (T_0 = 0, P_g =0). Their solution is more rigorous than presented by (Belokon 1959) and it yields $P_r/P_g \simeq 8.5$ for the condition of disappearance of shock jump. However, see Weaver & Chapline (1974), where it is again obtained $P_r/P_g \simeq 4.4$

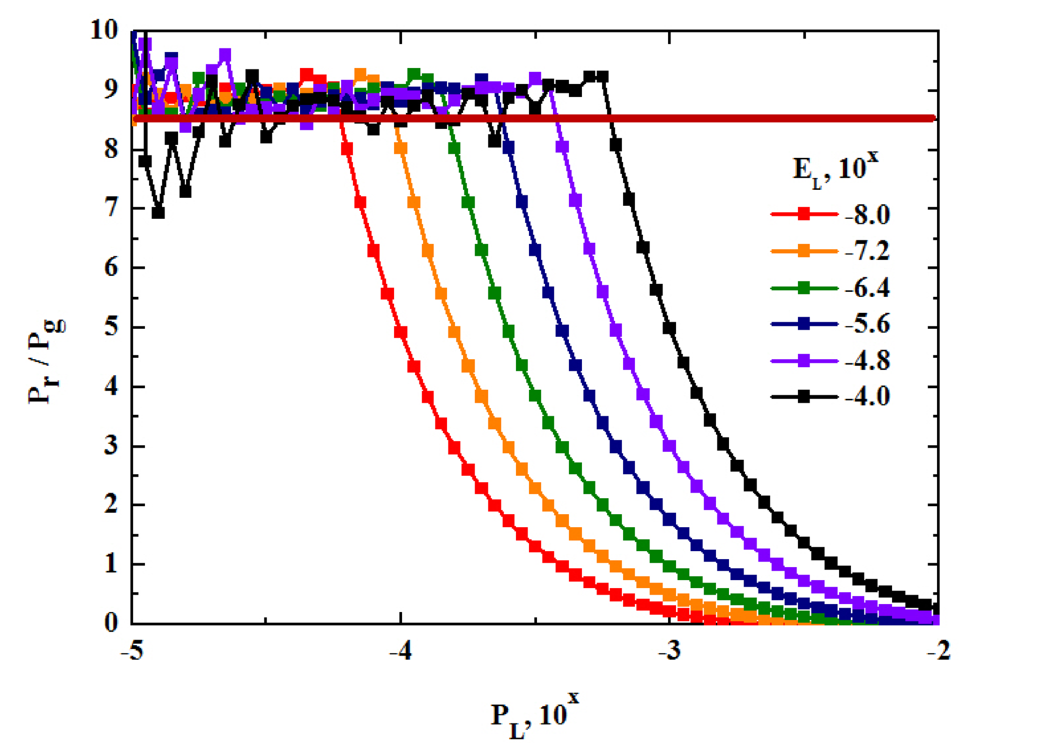


Fig.5. Disappearance of shock jump depending on shock tube configuration. Red solid line – theoretical value (Belokon 1959).

Our numerical analysis confirms the result $P_r/P_g \simeq 8.5$ for the condition of disappearance of viscous jump in case of non-relativistic shock waves. For relativistic case more accurate analyses must be performed beyond the critical point of velocity $v/c=(1/3)^{0.5}$

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FOR FUTHER INFORMATION

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